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11.1 - Introduction to Electric Potential and Electric Energy class="introduction"

Automated external defibrillator unit (AED) (credit: U.S. Defense Department photo/Tech. Sgt. Suzanne M. Day)

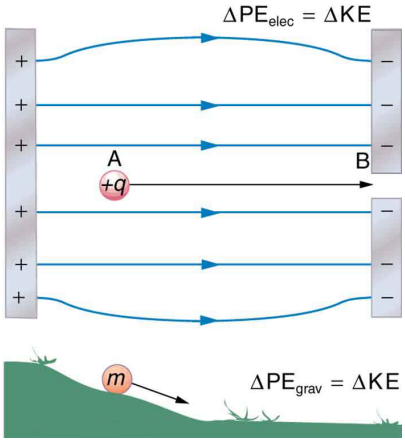
In Electric Charge and Electric Field, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of

electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

11.2 - Electric Potential Energy: Potential Difference

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge q is accelerated by an electric field, such as shown in [link], it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.



A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force

is conservative, we can write $W = -\Delta\text{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on *q* is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE, is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE. There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Note:
Potential Energy
$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE. There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation

without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = \mathrm{Fd}\cos\theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = \mathrm{qE}$, the work, and hence $\Delta\mathrm{PE}$, is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:
Equation:

$$V = \frac{\mathrm{PE}}{q}.$$

Note:
Electric Potential
This is the electric potential energy per unit charge.
Equation:

$$V = \frac{\mathrm{PE}}{q}$$

Since PE is proportional to *q* , the dependence on *q* cancels. Thus V does not depend on *q*. The change in potential energy $\Delta\mathrm{PE}$ is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where
Equation:

$$\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}} = \frac{\Delta\mathrm{PE}}{q}.$$

The **potential difference** between points A and B, $V_\mathrm{B} - V_\mathrm{A}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.
Equation:

$$1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

Note:
Potential Difference
The potential difference between points A and B, $V_\mathrm{B} - V_\mathrm{A}$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.
Equation:

$$1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by
Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

Note:
Potential Difference and Electrical Potential Energy
The relationship between potential difference (or voltage) and electrical potential energy is given by
Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example:
Calculating Energy
Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)
Strategy
To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge

through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta \text{PE} = q\Delta V$.
So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\ \text{C}$ and $\Delta V = 12.0\ \text{V}$. The total energy delivered by the motorcycle battery is

Equation:

$$\begin{aligned} \Delta \text{PE}_{\text{cycle}} &= (5000\ \text{C})(12.0\ \text{V}) \\ &= (5000\ \text{C})(12.0\ \text{J/C}) \\ &= 6.00 \times 10^4\ \text{J}. \end{aligned}$$

Similarly, for the car battery, $q = 60{,}000\ \text{C}$ and

Equation:

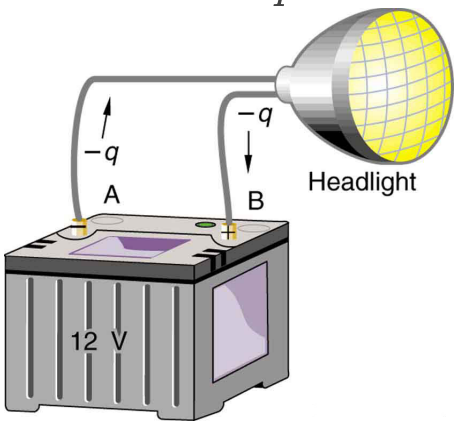
$$\begin{aligned} \Delta \text{PE}_{\text{car}} &= (60{,}000\ \text{C})(12.0\ \text{V}) \\ &= 7.20 \times 10^5\ \text{J}. \end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in [link]. The change in potential is $\Delta V = V_{\text{B}} - V_{\text{A}} = +12\ \text{V}$ and the charge q is negative, so that

$\Delta \text{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.



A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example:

How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta\text{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta\text{PE} = –30.0\ \text{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \text{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta\text{PE} = q\Delta V$:

Equation:

$$q = \frac{\Delta\text{PE}}{\Delta V}.$$

Entering the values for ΔPE and ΔV, we get

Equation:

$$q = \frac{–30.0\ \text{J}}{+12.0\ \text{V}} = \frac{–30.0\ \text{J}}{+12.0\ \text{J/C}} = –2.50\ \text{C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

Equation:

$$\text{n}_\text{e} = \frac{–2.50\ \text{C}}{–1.60\times10^{–19}\ \text{C/e}^–} = 1.56\times10^{19}\ \text{electrons}.$$

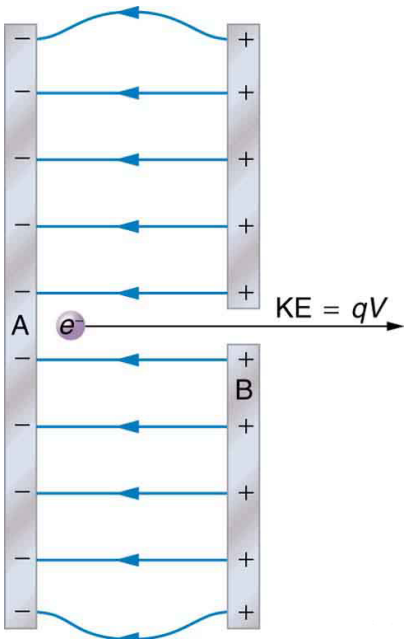
Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary

systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. [link] shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta \text{PE} = q\Delta V$, we can think of the joule as a coulomb-volt.



A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,
Equation:

$$\begin{aligned} 1\text{ eV} &= \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60 \times 10^{-19}\text{ J}. \end{aligned}$$

Note:
Electron Volt
On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,
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An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Note:
Connections: Energy Units
The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example,

calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30{,}000\ \text{eV} \div 5\ \text{eV per molecule} = 6000\ \text{molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as
Equation:

$$\text{KE} + \text{PE} = \text{constant}$$

or
Equation:

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f},$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example:
Electrical Potential Energy Converted to Kinetic Energy
Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)
Strategy
We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\mathrm{KE_i} = 0$, $\mathrm{KE_f} = ½\,mv^2$, $\mathrm{PE_i} = qV$, and $\mathrm{PE_f} = 0$.
Solution
Conservation of energy states that
Equation:

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Entering the forms identified above, we obtain
Equation:

$$qV = \frac{mv^2}{2}.$$

We solve this for v*:*
Equation:

$$v = \sqrt{\frac{2\mathrm{qV}}{m}}.$$

Entering values for q, V, and m gives
Equation:

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\text{ C}\right)\left(-100\text{ J/C}\right)}{9.11\times10^{-31}\text{ kg}}} \\ &= 5.93\times10^{6}\text{ m/s}. \end{aligned}$$

Discussion

Note that both the charge and the initial voltage are negative, as in [link]. From the discussions in Electric Charge and Electric Field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_{\text{B}}-V_{\text{A}}$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.
  Equation:

$$\Delta V = \frac{\Delta\text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,
  Equation:

$$\begin{aligned} 1\text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J}. \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\mathrm{KE} + \mathrm{PE}$. This sum is a constant.

Conceptual Questions

Exercise:

Problem:

What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

Exercise:

Problem: Voltages are always measured between two points. Why?

Exercise:

Problem:

How are units of volts and electron volts related? How do they differ?

Problems & Exercises

Exercise:

Problem:

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

Exercise:

Problem: Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car

from rest to 25.0 m/s, make it climb a 2.00×10^2 m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^2 N force for an hour.

Solution:

3.89×10^6 C

Glossary

electric potential
: potential energy per unit charge

potential difference (or voltage)
: change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

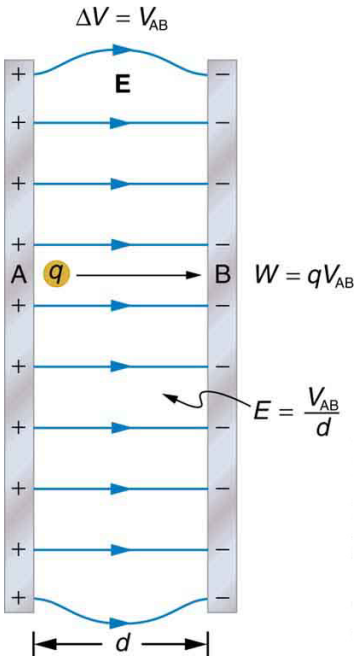
electron volt
: the energy given to a fundamental charge accelerated through a potential difference of one volt

mechanical energy
: sum of the kinetic energy and potential energy of a system; this sum is a constant

11.3 - Electric Potential in a Uniform Electric Field

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See [link].) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in Electric Potential Energy: Potential Difference, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.



The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{\text{AB}}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}$. See the text for details.)

The work done by the electric field in [link] to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

Equation:

$$W = -\Delta \mathrm{PE} = -q\Delta V.$$

The potential difference between points A and B is
Equation:

$$-\Delta V = -(V_{\mathrm{B}} - V_{\mathrm{A}}) = V_{\mathrm{A}} - V_{\mathrm{B}} = V_{\mathrm{AB}}.$$

Entering this into the expression for work yields
Equation:

$$W = qV_{\mathrm{AB}}.$$

Work is $W = Fd \cos\theta$; here $\cos\theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives
Equation:

$$qEd = qV_{\mathrm{AB}}.$$

The charge cancels, and so the voltage between points A and B is seen to be
Equation:

$$\left.\begin{array}{l} V_{\mathrm{AB}} = Ed \\ E = \frac{V_{\mathrm{AB}}}{d} \end{array}\right\} (\text{uniform } E \text{ - field only}),$$

where d is the distance from A to B, or the distance between the plates in [link]. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:
Equation:

$$1\ \mathrm{N/C} = 1\ \mathrm{V/m}.$$

Note:
Voltage between Points A and B
Equation:

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \frac{V_{\text{AB}}}{d} \end{array}\right\} (\text{uniform } E \text{ - field only}),$$

where d is the distance from A to B, or the distance between the plates.

Example:
What Is the Highest Voltage Possible between Two Plates?
Dry air will support a maximum electric field strength of about $3.0 \times 10^6 \text{ V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?
Strategy
We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{\text{AB}} = Ed$ can thus be used to calculate the maximum voltage.
Solution
The potential difference or voltage between the plates is
Equation:

$$\text{V}_{\text{AB}} = Ed.$$

Entering the given values for E and d gives
Equation:

$$V_{\text{AB}} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or
Equation:

$$V_{\text{AB}} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.

A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are

perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

Example:
Field and Force inside an Electron Gun
(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?
Strategy
Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{\text{AB}}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q\,\mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q\,E$.
Solution for (a)
The expression for the magnitude of the electric field between two uniform metal plates is

Equation:

$$E = \frac{V_{\text{AB}}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain
Equation:

$$E = \frac{25.0\ \text{kV}}{0.0400\ \text{m}} = 6.25 \times 10^5\ \text{V/m}.$$

Solution for (b)
The magnitude of the force on a charge in an electric field is obtained from the equation
Equation:

$$F = qE.$$

Substituting known values gives
Equation:

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^5\ \text{V/m}) = 0.313\ \text{N}.$$

Discussion
Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the

greater the electric field. In equation form, the general relationship between voltage and electric field is
Equation:

$$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Note:
Relationship between Voltage and Electric Field
In equation form, the general relationship between voltage and electric field is
Equation:

$$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

Section Summary

- The voltage between points A and B is
  Equation:

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \frac{V_{\text{AB}}}{d} \end{array}\right\} (\text{uniform } E \text{ - field only}),$$

where d is the distance from A to B, or the distance between the plates.

- In equation form, the general relationship between voltage and electric field is
  Equation:

$$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Conceptual Questions

Exercise:

Problem:

Discuss how potential difference and electric field strength are related. Give an example.

Exercise:

Problem:

What is the strength of the electric field in a region where the electric potential is constant?

Exercise:

Problem:

Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

Problems & Exercises

Exercise:

Problem:

What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $1.50 \times 10^4\ \mathrm{V}$?

Exercise:

Problem:

The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50 \times 10^4\ \mathrm{V/m}$. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

Solution:

(a) $3.00\ \mathrm{kV}$

(b) $750\ \mathrm{V}$

Exercise:

Problem:

(a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0 \times 10^6\ \mathrm{V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^3\ \mathrm{V}$ is applied? (b) How close together can the plates be with this applied voltage?

Solution:

(a) No. The electric field strength between the plates is $2.5 \times 10^6\ \mathrm{V/m}$, which is lower than the breakdown strength for air (

$3.0 \times 10^6\ \mathrm{V/m}$).

(b) 1.7 mm

Exercise:

Problem:

Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

Exercise:

Problem:

Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0 \times 10^6\ \mathrm{V/m}$.

Solution:

$15\ \mathrm{kV}$

Exercise:

Problem:

A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

Exercise:

Problem:

An electron is to be accelerated in a uniform electric field having a strength of $2.00 \times 10^6\ \mathrm{V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

Solution:

(a) 800 KeV

(b) 25.0 km

Glossary

scalar
: physical quantity with magnitude but no direction

vector
: physical quantity with both magnitude and direction

11.4 - Electrical Potential Due to a Point Charge

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge *q* from a large distance away to a distance of *r* from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential* V *of a point charge* is

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge)},$$

where *k* is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

Note:
Electric Potential V of a Point Charge
The electric potential V of a point charge is given by

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge)}.$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

Equation:

$$E = \frac{F}{q} = \frac{kQ}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

Example:

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb $(\mu\mathrm{C})$ range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a $-3.00\ \mathrm{nC}$ static charge?

Strategy

As we have discussed in Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = \mathrm{kQ}/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

Equation:

$$\begin{aligned} V &= k\frac{Q}{r} \\ &= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{-3.00 \times 10^{-9} \text{ C}}{5.00 \times 10^{-2} \text{ m}}\right) \\ &= -539 \text{ V}. \end{aligned}$$

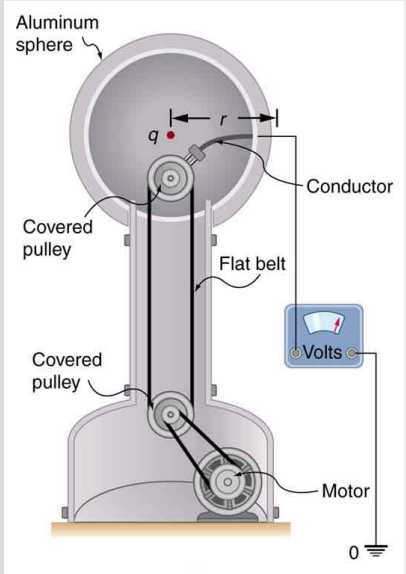
Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example:

What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See [link].) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)



The voltage of this demonstration Van de Graaff generator is measured

between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

Equation:

$$V = \frac{\mathrm{kQ}}{r}.$$

Solution

Solving for Q and entering known values gives

Equation:

$$\begin{aligned} Q &= \frac{\mathrm{rV}}{k} \\ &= \frac{(0.125\ \mathrm{m})\left(100\times10^{3}\ \mathrm{V}\right)}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}} \\ &= 1.39\times10^{-6}\ \mathrm{C} = 1.39\ \mu\mathrm{C}. \end{aligned}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in Electric Potential Energy: Potential Difference, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $\mathrm{PE_g = mgh}$.

Section Summary

- Electric potential of a point charge is $V = \mathrm{kQ}/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

Problems & Exercises

Exercise:

Problem:

A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

Solution:

144 V

Exercise:

Problem:

(a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

Solution:

(a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

Exercise:

Problem:

What are the sign and magnitude of a point charge that produces a potential of $-2.00\ \text{V}$ at a distance of 1.00 mm?

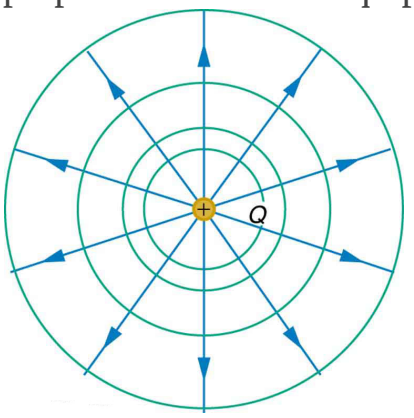
Solution:

$-2.22 \times 10^{-13}\ \text{C}$

11.5 - Equipotential Lines

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider [link], which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = \mathrm{kQ}/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of [link]. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.



An isolated point charge Q with its electric field lines in blue and equipotential lines

in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

Equation:

$$W = -\Delta \text{ PE} = -q\Delta V = 0.$$

Work is zero if force is perpendicular to motion. Force is in the same direction as , so that motion along an equipotential must be perpendicular to . More precisely, work is related to the electric field by

Equation:

$$W = Fd \cos \theta = qEd \cos \theta = 0.$$

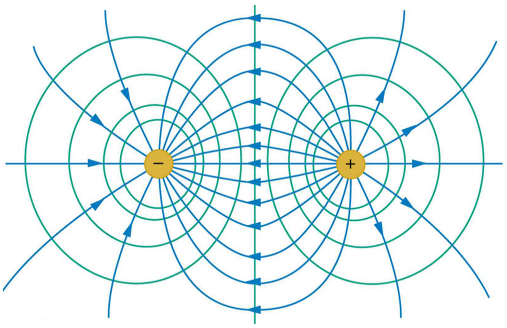
Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor nor d is zero, and so cos θ must be 0, meaning θ must be 90º. In other words, motion along an equipotential is perpendicular to .

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

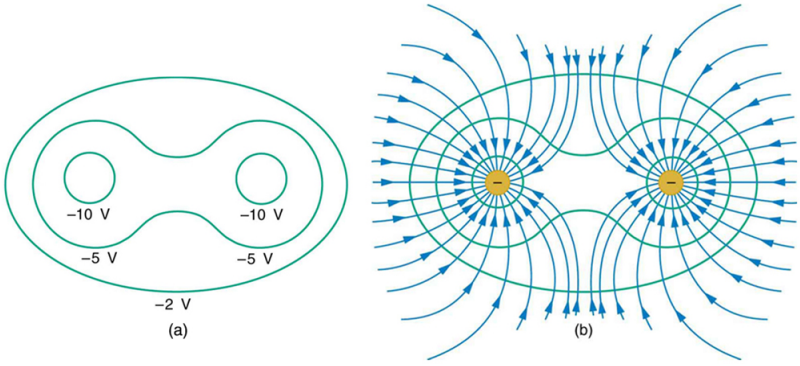
Note:
Grounding
A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in [link] a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

[link] shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in [link](a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in [link](b).



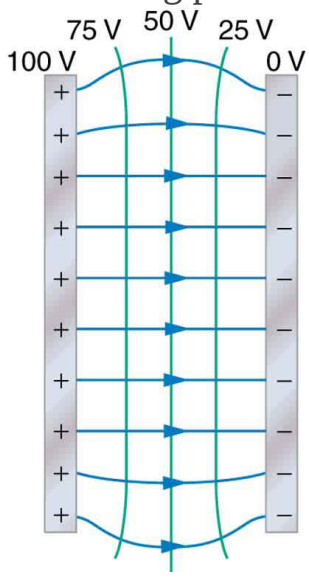
The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.



(a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them

perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in [link]. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.



The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a

defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in Energy Stored in Capacitors.

Section Summary

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

Conceptual Questions

Exercise:

Problem:

What is an equipotential line? What is an equipotential surface?

Exercise:

Problem:

Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

Exercise:

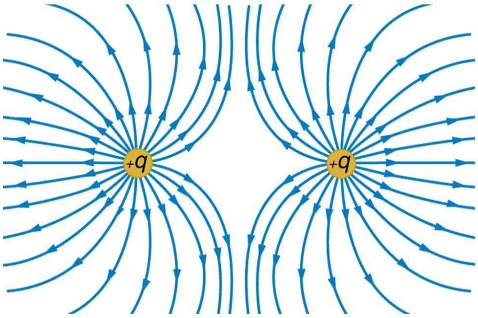
Problem: Can different equipotential lines cross? Explain.

Problems & Exercises

Exercise:

Problem:

Sketch the equipotential lines for the two equal positive charges shown in [link]. Indicate the direction of increasing potential.

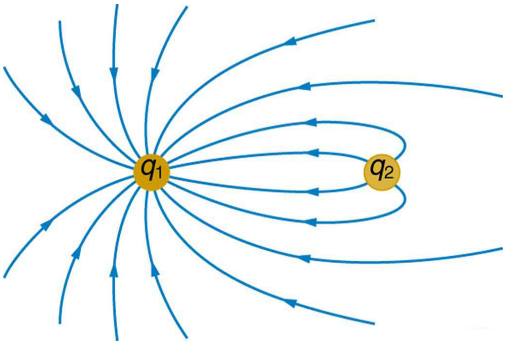


The electric field near two equal positive charges is directed away from each of the charges.

Exercise:

Problem:

Sketch the equipotential lines a long distance from the charges shown in [link]. Indicate the direction of increasing potential.



The electric field near two charges.

Glossary

equipotential line
: a line along which the electric potential is constant

grounding
: fixing a conductor at zero volts by connecting it to the earth or ground